# Multiscatter dark matter capture of our Sun

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Our Sun is one of the most resourceful candidates for providing evidence to direct scattering and accumulation of galactic dark matter. In this work, we explore the scattering between incoming galactic dark matter in both theoretical and computational approaches. We focus on the regime of DM particles scatter through the stellar particle medium. We also investigate heavy dark matter in particular, simulating the dark matter capture process using Monte-Carlo simulations with adaptations of FORTRAN package `swifter` to Python.

**Source code:** `https://github.com/kittenK531/starcode.git`

## I. INTRODUCTION

A usual approach to investigating known interactions between particles is by learning about the Lagrangian of the field and interactions. It remains an open question to particle physicists about where the well-developed boundaries lie upon all proposed models of dark matter. Understanding the particle nature opens up a new sector for Beyond Standard Model physics[1].

The first results of the direct detection of dark matter from LUX-ZEPLIN (LZ) have just been released[2]. Physicists have been putting a tremendous amount of effort into finding significant observable signatures for the detection of dark matter of specific predicted interactions from the Earth. Efforts in finding dark matter also by collider production is also another direct detection method in recent years[3]. It is, however, unfortunate that the terrestrial environment does not have a rich population of dark matter compared to galactic objects with long durations to have accumulated dark matter.

The most abundant population of dark matter is the large-scale gravitational structures accessed by astrophysical and cosmological observations. Where gravitational interactions in space provide us with a large sample of evidence to catch, and the limits only live within precision controls in the laboratories.

It is thus encouraging for physicists to probe the stellar environment for closer and larger stellar objects, which has been existing for billions of years and allow an abundant amount of dark matter to be captured and thermalized. Looking at closer stars lowers the bar to looking into signature observations when it comes to highly sensitive measurements, and removes the need to denoising the statistical background for observing far-away stars.

Without a fair estimation of dark matter profile in stars, it is rather hard to predict results from theoretical models, and the constraints on the detections cannot be anchored with confidence. But knowing how long the stars have been around helps physicists to make estimations for the amount of dark matter being trapped in the star, which also means it has accumulated a certain amount of dark matter to interact and be observed.

Many theoretical works of different proposed models have done estimations for respective capture rates, however, since there are no direct and certain conclusions from detections that can confirm the interaction between dark matter and other fields, it appears that the reality is more complex than the expectations of theoretical works. So our approach to starting to investigate dark matter capture is by simulating dark matter scatters trajectories with the limited interactions we can confirm, gravitational attraction.

## II. A THEORETICAL REVIEW OF THE MULTI-SCATTERING PROBLEM

This part reviews and derives the main quantities to be studied for the multi-scattering problem of incoming DM particles to the stellar mass. Cross-referencing the expressions from recent literature to set a solid foundation before carrying out the numerical Monte Carlo simulation for tracing scattering processes of DM. Here we introduce the scattering picture in two limits, the DM scattering with stellar fluid and stellar particles. We will discuss the fluid limit in brief under this section for the sake of completeness. We will have a more detailed and robust discussion on the particle regime in the next section, which is more significant to our simulation regarding the multi-scattering of the DM particle.

### A. Classical scattering in different limits

The kinetic theory illustrates collisions between DM particles in a medium in simple language. Using the mean free path or average distance to describe collisions between medium molecules of different regimes is a fair approach.

The mean free path denoted as $\ell_\chi$ can be characterized by the cross-sectional area[4][5] for collision in the medium.

Say a medium as above only contains one type of identical target mass $m_i$, that it contains total mass $M$ s.t.

$$M = Nm_i$$

where $N$ is the total number of target mass.

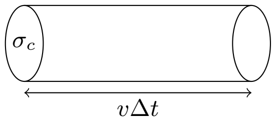


FIG. 1: Volume of medium crossed by cross-sectional area

In order to estimate the mean free path, it is essential to find out the number density of the target stellar mass.

$$n_i = \frac{N}{V} = \frac{M}{m_i}\frac{1}{V} = \frac{\rho_i}{m_i}$$

Recalling the mean free path refers to the distance of particle traveled without change of direction.

$$\ell_\chi \approx \frac{\text{length of path}}{\text{number of collisions}}$$

The number of collisions is approximately the same as the number of target mass inside the medium in the volume shown in the previous diagram.[6]

$$\ell_\chi \approx \frac{v\Delta t}{(\sigma_c \times v\Delta t)n_i} = \frac{1}{n_i\sigma_c} \tag{1}$$

### B. Classification of regimes

The quantity Knudsen number Kn (alternatively the optical depth $\tau_\star$) is defined as follows

$$\text{Kn} \equiv \frac{\ell_\chi}{2R_\star} \equiv \frac{1}{\tau_\star} \tag{2}$$

This is visually equivalence to having a cross-section of the star and estimating the number of collisions, to estimate the number density of the content (target mass) in the star.

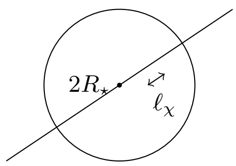


FIG. 2: Cross-sectional area of a spherical star with mean free path (Not to scale)

Considering the optical depth of DM particle, refers to how far the DM particle can see without bumping into or scattering from the molecules through the star. The smaller number of the mean free path that can fit into the diameter, implies the farther the DM particle can "see". And therefore, is more dilute, also requiring a larger number of Kn.

According to the article, Knudsen numbers below $\text{Kn} \lesssim 10^{-2}$ in a 75% hydrogen Population III star (assuming the target nuclei to be protons), corresponds to cross-sections $\sigma_c \gtrsim 10^{-34}\text{cm}^2$, regarded as the fluid regime. On the other hand, the Knudsen numbers $\text{Kn} \gtrsim 1$ corresponds to cross-sections $\sigma_c \lesssim 10^{-36}\text{cm}^{-2}$, regarded as particle regime.

### C. The Fluid Regime

From the previous sections, the average change in kinetic energy per distance traveled between scattering events $\ell_\chi$ is given as

$$\frac{\Delta E}{\ell_\chi} = n_i\sigma_c\Delta E. \tag{3}$$

In the fluid limit, it is relatively opaque to the DM particle, where $\ell_\chi \ll R_\star$ and $\text{Kn} \to 0$, under the scale of $R_\star$, consider energy along the scattering path, that is demonstrated in the figure as follows.

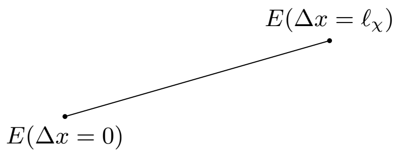


FIG. 3: Energy as a function of position in a head-on collision under classical approach (Not to scale)

Making use of Equation (3), for the third equality, the energy change per average scattering event distance is as follows

$$\frac{\Delta E}{\ell_\chi} = \frac{E(x_0+\ell_\chi) - E(x_0)}{\ell_\chi} \approx \frac{dE}{dx} \sim \frac{\rho_i}{m_i}\sigma_c\Delta E. \tag{4}$$

By requiring, $\Delta E = \Delta T$ which implies the microscopic potential energy (intermolecular potential energy) remains the same.

## III. MULTI-SCATTERING IN THE PARTICLE REGIME

In order to study the scattering for each encounter of dark matter particles to the baryonic stellar mass, we aim to trace consecutive classical scatterings with analytical methods. This problem is equivalent to studying the non-relativistic scattering between heavy cold dark matter and stellar mass.

To obtain the general expressions for the change of energy between consecutive collisions, the recoil velocity of dark matter becomes important. A simple elastic scattering condition was imposed to get the recoil velocity of dark matter in terms of initial condition parameters. By boosting any two-particle scattering frame to the center-of-mass frame, general analytical expressions are shown in detail.

### A. Non-relativistic kinematics in COM frame

Considering the only two scattering bodies, we can choose a particular frame s.t. both incoming particles align on the $y$-axis.

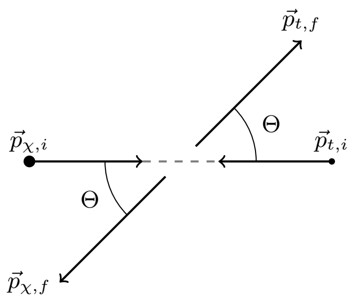


FIG. 4: Single classical scattering in center of mass frame

We further define the momenta of particles by the labeled schematic diagram above in Fig. 4, as the following expressions.

$$\begin{aligned}
\vec{p}_{\chi,i} &= M v_{cm,i}\left(0\ \ ,1\ \ ,0\right)\\
\vec{p}_{\chi,f} &= M v_{cm,f}\left(0\ \ ,-\cos\Theta\ \ ,-\sin\Theta\right)\\
\vec{p}_{t,i} &= m V_{cm,i}\left(0\ \ ,-1\ \ ,0\right)\\
\vec{p}_{t,f} &= m V_{cm,f}\left(0\ \ ,\cos\Theta\ \ ,\sin\Theta\right)
\end{aligned}$$

Imposing the elastic scattering condition,

$$\frac{|\vec{p}_{\chi,i}|^2}{2M} + \frac{|\vec{p}_{t,i}|^2}{2m} = \frac{|\vec{p}_{\chi,f}|^2}{2M} + \frac{|\vec{p}_{t,f}|^2}{2m} \tag{5}$$

$$\Rightarrow M\left(v_{cm,i}^2 - v_{cm,f}^2\right) = -m\left(V_{cm,i}^2 - V_{cm,f}^2\right) \tag{6}$$

With momentum conservation,

$$\begin{aligned}
&\left(M v_{cm,i} - m V_{cm,i}\right)\left(0\ \ ,1\ \ ,0\right)\\
&= \left(-M v_{cm,f} + m V_{cm,f}\right)\left(0\ \ ,\cos\Theta\ \ ,\sin\Theta\right)
\end{aligned}$$

By the center of mass, the initial momentum is zero implied $M v_{cm,i} - m V_{cm,i} = 0$. We further seek relations of the final momenta by using the zero initial momenta condition.

From the previous equation, the $y$-components showed

$$\begin{aligned}
M v_{cm,i} - m V_{cm,i} &= \left(-M v_{cm,f} + m V_{cm,f}\right)\cos\Theta\\
-M v_{cm,f} + m V_{cm,f} &= 0
\end{aligned}$$

, giving rise to

$$M v_{cm,f} = m V_{cm,f}. \tag{7}$$

Putting this condition to equation (5),

$$\left(M + m\left(\frac{M}{m}\right)^2 m\right) v_{cm,i}^2 = \left(M + m\left(\frac{M}{m}\right)^2\right) v_{cm,f}^2$$

yields

$$|v_{cm,i}| = |v_{cm,f}| \tag{8}$$

The total energy in the COM frame is

$$E_{cm} = \frac{1}{2} M v_{cm,i}^2 + \frac{1}{2} m V_{cm,i}^2 = \frac{1}{2}\left(M + \frac{M^2}{m}\right) v_{cm,i}^2 \tag{9}$$

### B. Non-relativistic kinematics in stellar rest frame

By boosting back to the stellar lab frame, the COM velocity is found to be

$$\dot{\mathbf{R}} = \frac{M\mathbf{v}_i + m\mathbf{V}_i}{M+m} \tag{10}$$

The energy in the lab frame, denoted with subscript $*$, is found as

$$\begin{aligned}
E_* &= E_{cm} + \frac{1}{2}(M+m)|\dot{\mathbf{R}}|^2\\
\frac{1}{2} M v_i^2 + \frac{1}{2} m V_i^2 &= \frac{1}{2}\left(M + \frac{M^2}{m}\right) v_{cm,i}^2 + \frac{1}{2}(M+m)|\dot{\mathbf{R}}|^2
\end{aligned}$$

Hence, it is possible to express the initial velocity of the incoming dark matter particle in the COM frame as

$$v_{cm,i} = \frac{\mu}{M}\sqrt{v_i^2 + V_i^2} = v_{cm,f}, \tag{11}$$

which is of the same magnitude as the final velocity of the dark matter particle.

With the final COM frame velocity defined as

$$\mathbf{v}_{cm,f} = \mathbf{v}_f - \dot{\mathbf{R}}, \tag{12}$$

the final velocity in the lab frame is found as

$$\mathbf{v}_f = \frac{\mu}{M}\sqrt{v_i^2 + V_i^2}\left(0\ \ ,\cos\Theta\ \ ,\sin\Theta\right) + \dot{\mathbf{R}} \tag{13}$$

Plugging in equation (10), and the squared modulus gives the final energy in lab frame,

$$\begin{aligned}
|\mathbf{v}_f|^2 &= |\dot{\mathbf{R}}|^2\\
&+ 2\frac{\mu}{M}\sqrt{v_i^2 + V_i^2}\left(\tfrac{M v_i^y + m V_i^y}{M+m}\cos\Theta + \tfrac{M v_i^z + m V_i^z}{M+m}\sin\Theta\right)\\
&+ \frac{\mu^2}{M^2}\left(v_i^2 + V_i^2\right)
\end{aligned}$$

Averaging the angular terms from $\Theta \in (0, 2\pi)$ removes the angular dependence on the average energy transfer per scattering.

### C. Continuous total energy loss of heavy DM particle in the particle regime

Whence the change of energy per scattering of dark matter particle in the lab frame is obtained as follows.

$$\begin{aligned}\Delta E &= \frac{1}{2}M\left(v_f^2 - v_i^2\right)\\ &= \frac{1}{2}M\left(|\dot{\mathbf{R}}|^2 + \frac{\mu^2}{M^2}\left(v_i^2 + V_i^2\right) - v_i^2\right)\\ &= \frac{1}{2}M\left[\left(\frac{M\mathbf{v}_i + m\mathbf{V}_i}{M+m}\right)^2 + \frac{\mu^2}{M^2}\left(v_i^2+V_i^2\right) - v_i^2\right]\end{aligned}$$

By expanding the terms and regrouping them corresponding to the initial velocities, the energy per scattering is obtained.

$$\Delta E = \frac{1}{2}M\left(-\frac{2\mu^2}{mM}v_i^2 + \frac{2\mu^2}{M^2}V_i^2 + \frac{2\mu^2}{mM}\mathbf{v}_i\cdot\mathbf{V}_i\right) \quad (14)$$

$$\approx -\frac{mM^2}{(m+M)^2}v_i^2 + \frac{m^2M}{(m+M)^2}V_i^2 \quad (15)$$

This shows a discrepancy to the calculation in Ellis's paper[7] by a factor of 2[8]. This resulting expression is later found to be consistent with the total energy loss for DM particle multi-scattering process from [9][10].

Considering stellar mass as particles, the total energy loss of the dark matter particle for n collisions can be expressed in this form.

$$\Delta E_{tot} = \sum_i (\Delta E)_i \quad (16)$$

Beginning with the first scattering process,

$$\Delta E_1 \approx -\frac{mM^2}{(m+M)^2}v_i^2 + \frac{m^2M}{(m+M)^2}V_i^2 \quad (17)$$

$$= \frac{\mu^2}{mM}\left(-Mv_i^2 + mV_i^2\right) \quad (18)$$

Recall that the energy loss of the incoming DM particle is also regarded as

$$\Delta E_1 = \frac{1}{2}M\left(v_1^2 - v_i^2\right) \quad (19)$$

Here, in order to simplify the expressions, for easy comparison, we adopt the convention from [10], which defines a new parameter as

$$\beta_\pm \equiv \frac{4Mm}{(M+m)^2}. \quad (20)$$

Equating the two energy loss per scattering expressions from (18), (22), the final velocity after this scattering is expressed in terms of the initial velocities.

$$v_1^2 = \left(1 - \frac{\beta_+}{2}\right)v_i^2 + \frac{2m^2}{(m+M)^2}V_i^2 \quad (21)$$

By generalizing the expression for the change of energy in intermediate scatterings,

$$\Delta E_j = \frac{1}{2}M\left(v_{j+1}^2 - v_j^2\right), \quad (22)$$

we can further simplify the expression to summing the total change of energies by noticing that $v_j$ is the final velocity of the $(j-1)$-th scattering and is the initial velocity of the $(j)$-th scattering.

We repeat the above treatment to obtain the magnitude of $v_j^2$ with equating $(j)$-th equations (18) and (22), deducing gives

$$v_j^2 = \left(1 - \frac{\beta_+}{2}\right)^j v_i^2 + \frac{2m^2}{(m+M)^2}\left(2 - \frac{\beta_+}{2}\right)^{j-1} V_i^2 \quad (23)$$

Whence, the total change of energy scattered away of an incoming DM particle after n collisions is given by

$$\Delta E_{tot} = \frac{1}{2}M\left(v_n^2 - v_i^2\right) \quad (24)$$

$$= \frac{M}{2}\left(\left(1-\frac{\beta_+}{2}\right)^n v_i^2 + \frac{2m^2}{(m+M)^2}\left(2 - \frac{\beta_+}{2}\right)^{n-1} V_i^2 - v_i^2\right) \quad (25)$$

$$\approx \frac{1}{2}Mv_i^2\left[\left(1 - \frac{\beta_+}{2}\right)^n - 1\right] \quad (26)$$

By assuming the DM particle is much heavier than the stellar particle, where $\mathcal{O}\left((m/M)^2\right) \ll 1$. The exact expression of the total loss of energy for the incoming DM particle after n-scattering is approximated and simplified.

### D. Comments on the total energy loss expression from recent literature

The expression of continuous scattering energy loss according to [10], also allows a probabilistic interpretation, where the fraction of energy loss in a single scatter is evenly distributed over the uniform distribution ranging from $|\Delta E|/E_i \in [0, \beta_+]$.[11] This also gives the average final energy per scattering as $(1 - \beta_+/2)E_i$. From this probabilistic picture, it is realized that

$$\Delta E_{tot} = E_f - E_i \approx E_i\left[\left(1 - \frac{\beta_+}{2}\right)^n - 1\right] \quad (27)$$

, converging to the same expression from our exact analytical result starting from the COM frame.

## IV. METHODOLOGY

Our goal is to trace the encountered DM particle in the sun following its trajectory under multi-scattering processes. For effective numerical integration to obtain the path traveled by the incoming DM particle, we adopted the swifter package written in Fortran[12] which greatly reduces the computational time for getting particle orbits around the star of interest using the symplectic fourth-order T+U method.

By integrating the swifter routine of the swifter, self-written scattering code following the theoretical background of the scattering picture from the previous sections is merged, and our Python module is developed for single particle scattering. This allows room for the development of further parallelization for multi-DM scattering pictures in order to investigate the capture process numerically. In the following sections, we will focus on two main parts, the work to adapt swifter to a Python-compatible environment, and the self-developed algorithm for the multi-scattering processes.

### A. A brief introduction to Swifter

Swifter is a FORTRAN written software that simulates many-body scattering problems numerically. With the initialization of the Sun located at the origin, we can alter the information of planets and the number of test particles by modifying two of the input files, `pl.in` and `tp.in` respectively. We specify the mass, position, and velocity of the planets, also the initial conditions for the test particles, via tuning the integration parameters in the text file `param.in`, and running the executable `swifter_tu4`, the position and velocity information of the targeted test particle will be integrated and encoded in a binary file. By executing another binary `tool_follow`, the corresponding resulting binary is then decoded into a human-readable text file.

Swifter is an integrator that adopts the unit set requiring the gravitational constant $G$ to be unity. Here we choose the unit set of length in $AU$, and time in $day$.

### B. Adaptation of swifter

In order to make use of swifter as subroutines, we call the binaries in our python running environment using the Python-friendly package of `subprocess`[13]. The main goal is to specify our working directory and run our binaries smoothly to output NumPy arrays of any desired dimension. For each evolution of the incoming DM particle, we use bash editing to change its corresponding initial conditions in the input file `tp.in` and read out the corresponding numerical values for both the position and velocities in heliocentric coordinates using solely Python.

To avoid confusion and mixing of input files, for each evolution, we create a new input file with a file name of a newly generated hash[14] by duplicating the sample input file. We then amend the values following the format of writing the input files, with discriminators as a particular number of spacing. Since we do not work in the shared memory `/dev/shm` directory, it is more memory efficient to disregard and delete the input and output files from previous runs before each run starts.

We also notice that the display of certain numbers in the output text file is mislabelled as a chain of asterisks, hence we suspected it was a display malfunction for FORTRAN and moved on to debug and recompile the software. The detailed operation can be found in the public GitHub repository link provided in my abstract.

This adaptation written in python helps us to get the coordinates and velocities in a shorter computational time and is a safer means of carrying out symplectic integration. We will discuss the convergence testing and sanity checks for continuous input and output files for subsequent evolution.

### C. Scattering algorithm

In our simulation, we assume our Sun is an isotropic medium, where the density has no radial dependence. We only assume the isotropic ball has 75% Hydrogen and 25% Helium in terms of atomic composition.

Our scattering algorithm has two main parameters to vary, one is the DM mass, another is the scattering cross-section, or in other words, the interaction strength. The flowchart is shown in Figure (5), which indicates that this software relies mainly on three while loops. The first is under the condition of which the DM particle is yet not captured by the Sun, wrapping the other two while loops, that force the DM to orbit the Sun until entering it and scattering through the medium until the energy of the DM particle is smaller than the escape energy while being inside the Sun.

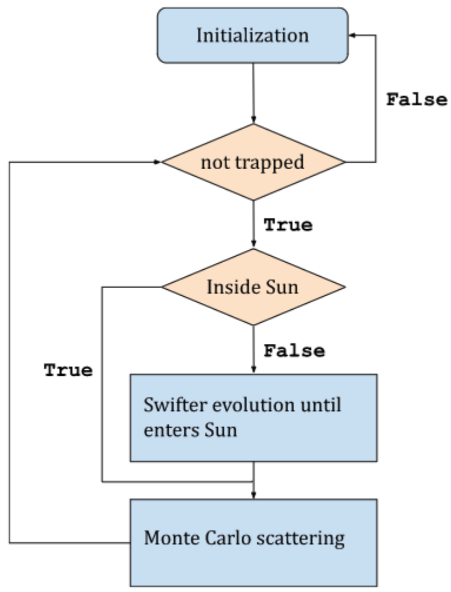


FIG. 5: Flowchart of `starcode`

## D. Dark Matter Capture Implementation logic

Since we have discussed the swifter implementation in detail in the previous subsection, we will focus on the developments made on our own. After being able to read out the position and velocities in NumPy arrays, we move to check if the particle is inside the star and whether the DM particle has sufficient energy to escape. Fulfilling both being in the star and the particle having speed smaller than the escape velocity (magnitude), we conclude the dark matter is being captured.

For initialization, we randomly pick a velocity under the Maxwell-Boltzmann distribution curve. With the knowledge that the number flux of DM particles on a star, according to Ellis[7], follows

$$\Phi_{DM} = \pi R_{\odot} n_{DM} \int d^3 v f(\vec{v}) v \left(1 + \frac{v_{esc}^2}{v^2}\right) \tag{28}$$

, where $f(\vec{v})$ is the Maxwellian velocity distribution, $n_{DM}$ denotes the number density of DM and $v$ is the velocity of the DM particles.

### *1. Restoring statistics of Maxwell-Boltzmann distribution for DM particle velocities*

Our work to obtain the velocity for a DM particle under a Maxwell-Boltzmann distribution

$$f(\vec{v}) = \sqrt{\left(\frac{m}{2\pi kT}\right)^3} 4\pi v^2 e^{-\frac{mv^2}{2kT}} \tag{29}$$

is based on a mathematical technique called Box-Muller transform, where each axis velocity element should satisfy the scaled gaussian distribution as Maxwellian is.

Using Box-Muller transform, we transform uniformly distributed independent random variables $x_1$, $x_2$ to a standard normal distribution variable $y$ by

$$y = \sqrt{-2 \ln x_1} \cos(2x_2) . \tag{30}$$

We then obtain the velocity components by multiplying the variable $y$ to a scaling factor dependent on the particle energy

$$v_i = \sqrt{\frac{kT}{m}} \times y \tag{31}$$

, note that we use natural units in our calculations i.e. $G = c = \hbar = k = 1$.

After initialization, we move on the give the DM particle an initial kick with the initial Box-Muller transformed velocity so that it evolves using swifter, looping until it enters the Sun. Once it enters the sun, we record the position and velocity upon encounter, and we visualize the entry point for sanity-checking purposes. We then move on to the last while loop mentioned a few paragraphs ago.

### *2. Energy change in a probabilistic picture*

After entering the Sun, we make use of our cross-section of interest and evaluate it according to the mean free path upon scattering as derived in Equation (1). We implement the bounded uniform distribution of $|\Delta E|/E_i \in [0, \beta_+]$ giving us both possibilities of incoming DM particle gain and loss energy via scattering with the stellar medium. Therefore, this yields final energy upon scatter, and we simply promote our drawn mean free path, from a uniform distribution of $\lambda \in [0, 2\ell_\chi]$ s.t. the average gives $\bar{\lambda} = \ell_\chi$, from the position upon entry to a randomized direction of a magnitude of drawn mean free path.

### *3. Randomization of directions after scattering*

The randomization of both velocity and position directions (unit vectors) are identical after each scatters according to Figure (3). This randomization relies on the background that any vectors are projected according to the unit vector orientations. In this work, we chose to generate two independent random numbers $u$, $v$ both in the range of $[0, 2\pi]$ s.t. the projected quantities are given by

$$\mathbf{r} = |\mathbf{r}| \left(\cos u \sin v \quad , \sin u \sin v \quad , \cos v\right) \tag{32}$$

## E. Visualization

In this subsection, we wish to include the technique and development highlights to visualize this simulation. From Figure (6) and (7), we obtained information for

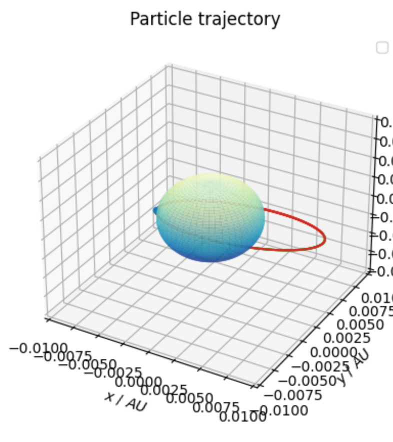


FIG. 6: DM trajectory emphasized the first encounter position at the Sun surface with complete orbit

the incoming DM particle upon entry for orbits that exceeded our amount of use where after entry, the numer-

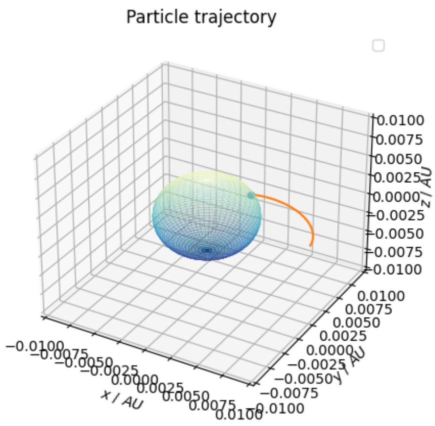


FIG. 7: DM trajectory emphasized the first encounter position at the Sun surface with orbit only before encounter

ical values are kept. However, since we would like to observe the whole orbiting and scattering process where we disabled visualizing the orbit position after entry to avoid confusion. In Figure (8), we observe one scattering event of the DM being scattered 6 times with cross-section $\sigma_c = 5.95 \times 10^{-35} cm^2$ and escaping through the Sun's surface. This illustrates our visualization is made very human-friendly for inspecting the simulation results.

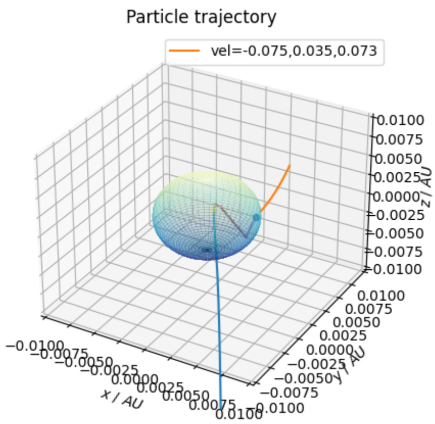


FIG. 8: DM trajectory emphasized the first encounter position at the Sun surface with orbit motions and scattering

## V. RESULTS AND DISCUSSION

In this work, we successfully simulated single dark matter capture events with the implemented logic mentioned in the previous section. In this section, we will discuss several dark matter capture events.

### A. Monte Carlo simulation features

By fixing the initial position and velocity for the first kick, we can observe different scattering events more closely, we intentionally chose a relatively small velocity so that it gets captured in a few scattering events.

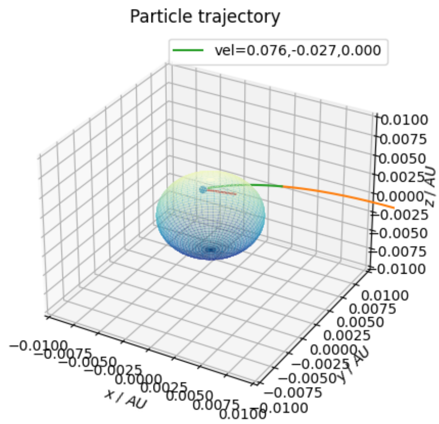


FIG. 9: Type I: Captured dark matter after one scatter event, with $\sigma_c = 5.95 \times 10^{-35} cm^2$ and $m = 1000 GeV$

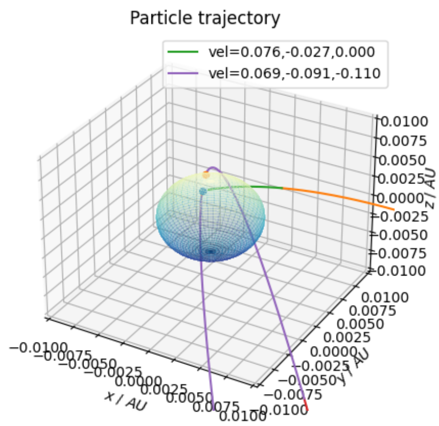


FIG. 10: Type II: Captured dark matter after multiple scatter events, with $\sigma_c = 5.95 \times 10^{-35} cm^2$ and $m = 1000 GeV$

From the results in Figure (9), and (10), we can see the Monte Carlo simulation property of the randomized results is displayed with fixed initial conditions. Types of results can be shown to be captured within one scatter event and in several scatter events.

## B. Scattering times per scattering events and cross-section

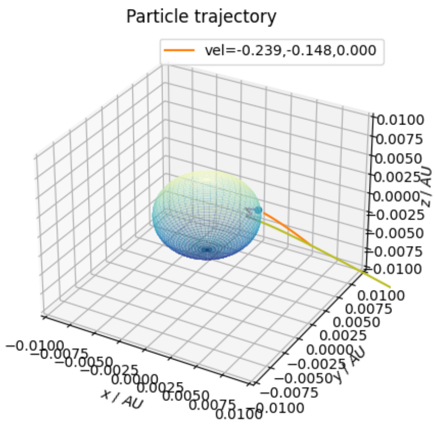


FIG. 11: Dark matter escaped in one scattering event which scattered 4 times, $\sigma_c = 1.84 \times 10^{-34} cm^2$ and $m = 1000 GeV$

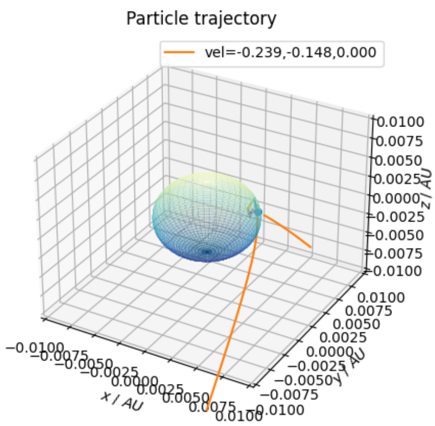


FIG. 12: Dark matter escaped in one scattering event which scattered 17 times, $\sigma_c = 1.84 \times 10^{-34} cm^2$ and $m = 1000 GeV$

From a very little sample size, we observe that DM scatters more times per scattering event before leaving the Sun when we have larger values of the cross-section. This makes sense since larger cross-sections mean stronger interaction strength between the incoming DM particle and stellar particles.

## C. Convergence and sanity checking

In this section, we look into the sanity checkings we performed to ensure our simulation results matches our expectations.

### 1. *Swifter: I/O continuity of subsequent runs*

Since we amend the input files every iteration to obtain one revolution, it is our baseline to ensure that the next revolution picks up the final kinematic information correctly and continues propagating. Figure (13) shows that

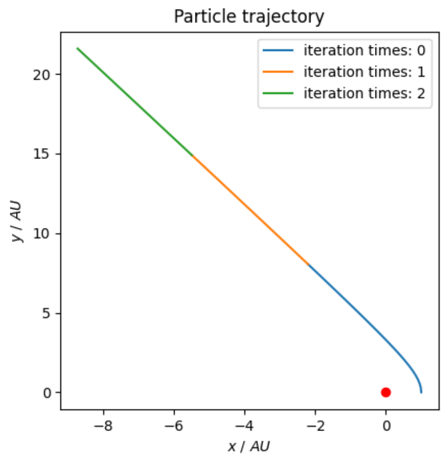


FIG. 13: Dark matter trajectory about the Sun as continuous, gap-less propagation

the DM trajectory is continuous and propagates from the previous position to the next. Here we ensured each Keplerian iteration is physical.

### 2. *Continuous Keplerian orbits with encounter and without encounter*

In this subsection, we wish to check if the orbits are symplectic, continuous, and reasonable after our modification of logic for the software to tell if the DM particle enters the Sun.

For closed orbits which the DM particle does not enter the Sun,
from Figure (14), the orbits completely overlap with one another implies the integration done by swifter is indeed energy-conserving. The second point to check here is, not entering the Sun, the DM particle can never lose energy in any other way, which makes the DM particle unable to enter the Sun forever.

In Figure (15), we observe a successful run for a DM particle to continue one's trajectory until encountering the Sun, which is the second swifter iteration here.

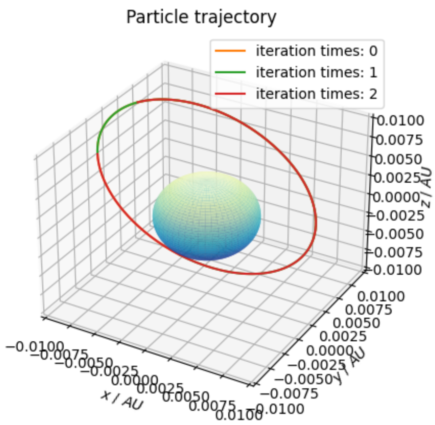


FIG. 14: Dark matter continuous symplectic path as closed orbits

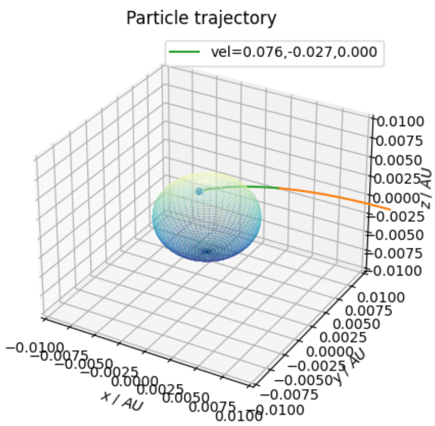


FIG. 15: Dark matter particle encounter on second swifter run

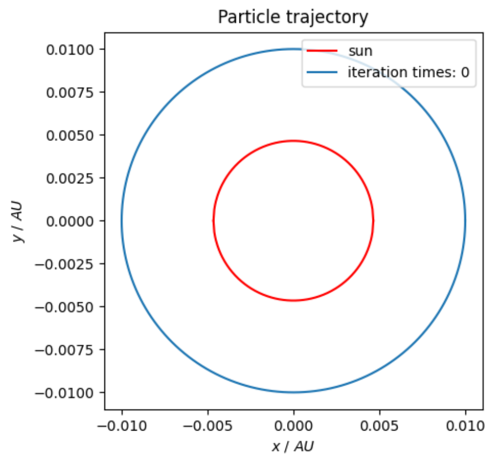


FIG. 16: Dark matter escaped in one scattering event which scattered 17 times, $\sigma_c = 1.84 \times 10^{-34} cm^2$ and $m = 1000 GeV$

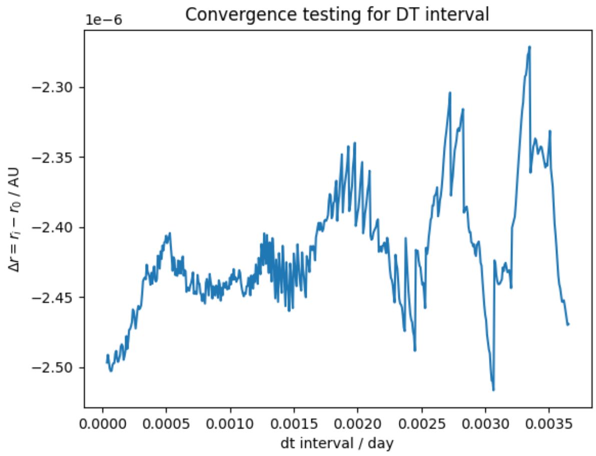


FIG. 17: Dark matter escaped in one scattering event which scattered 17 times, $\sigma_c = 1.84 \times 10^{-34} cm^2$ and $m = 1000 GeV$

### *3. Convergence testing*

The last test to check is whether swifter shows convergence upon reduction of integration step size. In order to carry out the convergence test, we find the variable $r$ which is the radius of a circular orbit around the Sun, which is a stable fixed parameter as shown in Figure (16). By fixing the DM initial conditions identical to the velocity of the Earth, we evaluate the fluctuation for integration with bigger step size and obtained the graph of Figure (17).
From Figure (17), reading from large values of $dt$ to smaller ones, we see the amplitude of fluctuations decreases rapidly, passing our convergence test.

## VI. ISSUES AND IMPROVEMENTS

One main improvement to be made is rewriting or having a FORTRAN wrapper in Python. This is the means that allow efficient parallelization to be done when it comes to multi-particle scattering. Another improvement we have in mind is to disable the stdout from the original swifter program since printing out the parameters for integration for every orbit event is time-consuming.

## VII. CONCLUSION AND FUTURE WORK

In this work, we reviewed the theoretical DM scattering in the particle regime and re-derived the expression for total energy loss upon continuous scattering which converges with the recent literature results. We also showed successful dark matter trapping events by employing the modified swifter software and passed relevant sanity checks successfully.

In the future, we wish to further improve our software and investigate the thermalization time and hopefully could gather enough data to look into the relationship between interaction strength and dark matter mass. We also hope to further upgrade our code to allow parallelization in order to perform multi-DM multi-scattering on clusters. We also hope to extend this pipeline to a number of known astrophysical bodies, by obtaining important quantities such as the scattering cross sections to dark matter mass relationships, we can further identify our most optimal targets which cover a range of interaction models for dark matter.

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[4] Often in kinetic theory we regard this as an effective collision area. Since we are considering DM particle collision with different molecules like Hydrogen and Helium, it is still a variable considering which case it lies in.
[5] More specifically speaking, in scattering language, $\sigma_c \equiv \frac{\text{Number of particles scattered per atom per sec}}{\text{Number of beam particles per } cm^3 \text{per sec}}$.
[6] $v$ is the exact velocity before each collision.
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[13] I have to thank Dr. TSE Kin Fai for his idea on facilitating the running of the binaries without calling OS.
[14] A random string of six digits.